1983 Q1

A train of length 120m has an acceleration of $1 m/s^2$. It meets another train of length 80m travelling on a parallel track in the opposite direction with an acceleration of $1.5 m/s^2$. Their speeds at this moment are respectively 20m/s and 25m/s. Show, by diagrams, the positions of the trains just before meeting and immediately after passing.
Find the time taken for the trains to pass each other. **(4secs)**
If one of the trains, by applying breaks, were to cause an increase of $12\frac{1}{2}$ % in this time of passing, calculate to the nearest m/s^2 the increase in its acceleration. **($3 m/s^2$)**

1983 Q2

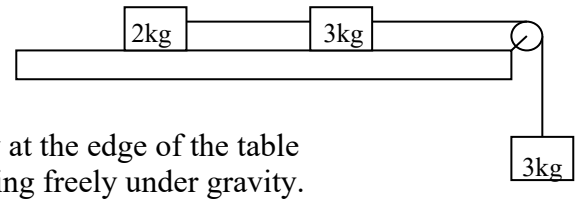
An aircraft flew due east from p to q at u_1 km/h. Wind speed from the south west was v km/h. On the return journey from q to p, due west, the aircraft's speed was u_2 km/h, the wind speed being unchanged. If the speed of the aircraft in still air was x km/h, $x > v$, show, by resolving along and perpendicular to pq, or otherwise, that

(i) $u_1 - u_2 = v\sqrt{2}$

(ii) $u_1 u_2 = x^2 - v^2$

If $|pq| = d$, find in terms of v, x and d, the time for the two journeys.

The diagram shows particles of mass 2kg and 3kg, respectively lying on a horizontal table in a straight line perpendicular to the edge ot the table. They are connected by a taut, light, inextensible string.



A second such string passing over a fixed, light pulley at the edge of the table connects the 3kg particle to another of mass 3kg hanging freely under gravity.
The contact between the particles and the table is rough with coefficient of friction ¼. Show in separate diagrams the forces acting on the particles when the system is released from rest. Calculate

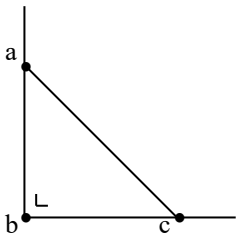
(i) the common acceleration

(ii) the tension in each string in terms of g.

A bullet is fired from a gun fixed at a point o with speed v m/s at an angle of θ to the horizontal. At the instant of firing, a moving target is 10m vertically above o and travelling with constant speed $42\sqrt{2}$ m/s at an angle of 45^0 to the horizontal. The bullet and target move in the same plane.
If $v = 70$ m/s and $\tan\theta = \frac{4}{3}$, find at what time after firing does the bullet strike the target and calculate the horizontal distance of the bullet from o.
Show that the *only* value of θ to ensure that the bullet strikes the target is given by $\tan\theta = \frac{4}{3}$

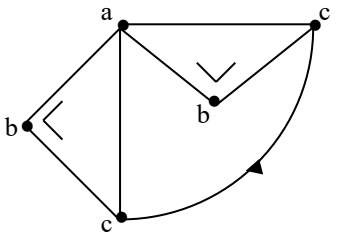
A State the laws governing oblique collisions between two smooth elastic spheres.
Two such spheres A and B of mass 5 and 10 kg respectively, collide obliquely. The coefficient of restitution is $\frac{1}{7}$.
Immediately before collision the velocity of A is $5\vec{i} + 4\vec{j}$ and that of B is $-2\vec{i} - 2\vec{j}$, where speeds are in m/s and $\vec{i}$ and $\vec{j}$ are unit vectors along and perpendicular to the line of centres.
Find the velocity of (i) A and (ii) B after impact. Show that the loss of kinetic energy is 80J.
Calculate the tan of the angle through which B is deflected after the collision.

(i) A uniform triangular lamina *abc* is of mass *m* with $|ab| = |bc| = 6$ and $|\angle abc| = 90^0$. Show that its moment of inertia about *bc* is 6*m*.



(ii) Prove that the moment of inertia of *abc* about an axis through *a* perpendicular to the plane of *abc* is 24*m*. [Coordinates of *g*, the centre of gravity of *abc* is (2,2) when the origin is at *b*]

(iii) The axis through *a* is fixed horizontally so that the lamina can rotate freely under gravity in a vertical plane. It is released from rest with *ac* horizontal and above *b*. Find in terms of g, the speed of *c* when *ac* is vertical.



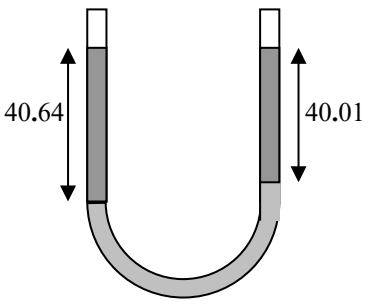
A hollow right circular cone of semivertical angle α where $\tan\alpha = \frac{3}{4}$ is fixed with its axis vertical and vertex downwards. The inner surface of the cone is rough with coefficient of friction ½ and the cone rotates about its axis with uniform angular velocity 7 rad/s.
A particle of mass *m* is placed on the inside surface and rotates with the cone at a vertical height *h* above the vertex. Calculate the normal reaction of the particle with the inside surface and the height *h* above the vertex if
(i) the particle is about to slide down
(ii) the particle is about to slip up.

Define simple harmonic motion in a straight line and show that $x = a\sin\omega t$ can describe such motion, when *x* is the distance from a fixed point and *a*, ω and *t* have the usual meanings.
A particle *p*, of mass 5kg, is connected by a light elastic string, of natural length 2m and elastic constant 140 N/m to a fixed point *q* on a rough horizontal surface where the coefficient of friction is 1. *p* is released from rest at a point a where $|qa| = 3$m.

By considering the forces acting on *p* when its distance is $(2.35 + x)$m from *q*, prove that *p* moves in simple harmonic motion as long as the string remains taut. State the position of the centre, *o*, of the simple harmonic motion i.e. $|qo|$ and write down the amplitude.

If the periodic time is assumed to be $\frac{\pi}{\sqrt{7}}$ calculate the time taken by the particle to travel from *a* to a point 2m from *q*.

(a) Mercury occupies the curved portion of a fixed upright U-tube of uniform cross-section. Water is poured into one arm and alcohol into the other, until both free surfaces are at the same level. The lengths of water and alcohol columns are 40.64cm and 40.01cm respectively. Calculate the density of alcohol. More alcohol is then poured in until the two surfaces are at the same level. Find the new length of the alcohol column to the nearest mm.
[Density of water = 1000 kg/m^3. Relative density of mercury = 13.6]



(b) A right circular cone of base radius *r* and vertical height 3*r* is held submerged with its vertex downwards in a liquid of density ρ, its plane base is horizontal and is at a distance *r* below the surface. Calculate the forces exerted by the liquid on
(i) the base
(ii) the curved surface of the cone.

(a) Find the solution of the differential equation $\sin x \frac{dy}{dt} = y\cos x$ when $y = 2$ at $x = \frac{\pi}{6}$.

(b) A particle of mass 8kg moves along a line (the x-axis) on a smooth horizontal plane under the action of a force in newtons of $(40 - 3\sqrt{x})\vec{i}$ where $\vec{i}$ is the unit vector along the axis and x is the displacement of the particle from a fixed point o of the axis. If the particle starts from rest at o, find its speed when $x = 100$ and calculate where it next comes to instantaneous rest.

$\left[acceleration = v\frac{dv}{dx} \right]$