1987 Q1

(a) The maximum acceleration of a body is 4 m/s^2 and its maximum retardation is 8 m/s^2. What is the shortest time in which the body can travel a distance of 1200m from rest to rest ? **(30s)**

(b) A car, *A*, starts from a point *p* with initial velocity of 8m/s and then travels with a uniform acceleration of 4m/s^2. Two seconds later a second car *B* starts from *p* with an initial velocity of 30m/s and then moves with a uniform acceleration of 3m/s^2. Show that after passing *A*, *B* will never be ahead by more than 74m.

1987 Q2

At a certain instant a ship *H* is 37.5km due West of a ship *K*. Ship *H* is travelling South-East at 25 km/h and ship *K* is travelling South at 15 km/h.

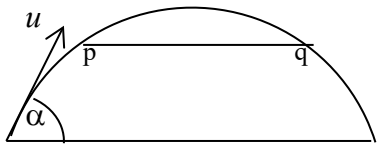
(i) Draw a diagram to show the velocity of *K* relative to *H* and calculate the magnitude and direction of the relative velocity.

(ii) If *H* and *K* can exchange signals when they are not more than 20km apart, calculate when they can begin to exchange signals and for how long they can continue to exchange signals.

1987 Q3

(a) A particle is projected up a plane, which is inclined at an angle $\tan^{-1}(¼)$ to the horizontal. The direction of projection makes an angle α with the inclined plane. (The plane of projection is vertical and contains the line of greatest slope). If the particle strikes the inclined plane at right angles, show that $\tan \alpha = 2$.

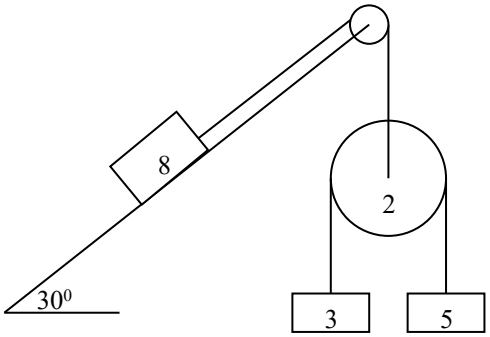
(b) A particle is projected with speed *u* at an angle α to the horizontal. The particle takes 4s to travel between two points *p* and *q* which are on the same horizontal line. Show that the greatest height the particle reaches above this line is 19.6m.



1987 Q4

A particle of mass 8kg rests on a rough plane which is inclined at 30^0 to the horizontal. The coefficient of friction between the particle and the plane is $\frac{1}{\sqrt{3}}$.

The 8kg mass is connected by a light inextensible string passing over a smooth fixed pulley at the top of the plane, to a pulley of mass 2kg hanging freely. Over this pulley which is also smooth, a second light inextensible string is passed having particles of mass 3kg and 5kg respectively, attached.



(i) Show in a diagram the forces acting on each mass when the system is released from rest.

(ii) Calculate the acceleration of the 8kg mass.

1987 Q5

State the laws governing the oblique collision of smooth elastic spheres.

Two smooth elastic spheres *A* and *B* of mass 4kg and 8kg respectively, collide obliquely. The coefficient of restitution is 0.4. Before collision the velocity of *A* is $(3\vec{i} + 4\vec{j})$ m/s and that of *B* is $(-4\frac{1}{2}\vec{i} - p\vec{j})$ m/s where $\vec{i}$ and $\vec{j}$ are unit vectors along and perpendicular to the lines of centres at the moment of impact

(i) Find the velocity of each sphere after impact

(ii) Show that the loss of kinetic energy, as a result of the impact is 63J

(iii) If after impact the spheres are moving at right angles to each other calculate the value of p.

1987 Q6

Define simple harmonic motion.

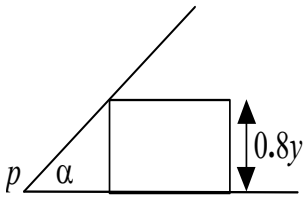
A particle of mass *m* is suspended from a fixed point *p* by a light extensible string of natural length *d* and elastic constant $\frac{49m}{d}$. It is pulled down a distance $\frac{8d}{5}$ below *p* and is then released from rest.

(i) Show that the particle moves with simple harmonic motion as long as the string remains taut.

(ii) Find in terms of *d*, when the string becomes slack for the first time.

1987 Q7

Define limiting friction and coefficient of friction.
A uniform rod of mass 2 kg and of length $6y$ metres, leans against the smooth edge of a rectangular block of mass 6 kg and height $0.8\,y$ metres. The rod is smoothly hinged at p to a rough horizontal floor and the block also rests on the floor (see diagram). The block is on the point of slipping when the rod makes an angle α with the horizontal, where $\tan\alpha = \frac{4}{3}$.



(i) Show in separate diagrams the forces acting on the rid and on the block.

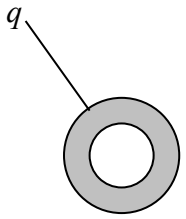
(ii) Show that the coefficient of friction between the block and floor is $\frac{6}{17}$

Find, correct to the nearest Newton, the magnitude of the reaction at the hinge.

1987 Q8

Prove that the moment of inertia of a uniform annulus of internal diameter p, external diameter $3p$ and mass $4m$, about an axis through its centre perpendicular to its plane is $5mp^2$. (see tables P.40)

A uniform rod of mass m and length $6p$ is attached to the rim of this annulus so that the rod and the annulus are in the same plane and the rod is collinear with a diameter of the annulus (see diagram). If the compound body is set in motion about an axis through q which is perpendicular to the plane of the rod and the annulus,



(i) find the period of small oscillations.

(ii) show that the length of the equivalent simple pendulum is $\frac{22p}{3}$.

1987 Q9

(a) A wooden cube of side 10cm, and relative density 0.8, is floating horizontally in water. What mass of aluminium, whose relative density is 2.8 must be attached to

(i) the upper surface, so that the cube will just be completely immersed horizontally with the aluminium above water?

(ii) the lower face, so that the cube is just immersed and horizontal.

(b) A uniform rod in equilibrium is inclined to the horizontal with one fifth of its length immersed in a liquid and its upper end supported by a vertical force P.

(i) Show in a diagram the forces acting on the rod.

(ii) If the relative density of the rod is 0.72, calculate the relative density of the liquid.

1987 Q10

(a) Solve the differential equation $2x(1+y)\dfrac{dx}{dy} = 8 + x^2$ if $x = 2$ when $y = 3$

(b) The resistance to motion of a train of mass m is constant and equal to 60N per tonne. When moving with constant speed 16 m/s on a level line the train begins to ascend an incline of 1 in 98, ie $\sin^{-1}(1/98)$. Assuming that the engine continues to work at the same rate (ie power is constant) and that v m/s is the speed of the train up the incline t seconds after the train has begun to climb, show the equation of motion is $\left(\dfrac{v}{v-6}\right)\dfrac{dv}{dt} + \dfrac{4}{25} = 0$

Calculate the time which elapses before the velocity falls to 12 m/s.
(Tables P.29 may be needed)